Information-theoretic Limitations of Data-based Price Discrimination

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Price discrimination

- a seller selling a product to buyers
- buyers have willingness to pay Y
- ... and a one-dimensional covariate X
- uniform pricing: single posted price for every buyer
- (third-degree) price discrimination: set different price for different realization of $X = x$

Traditional pricing theory

- joint distribution $F_{Y,X}$ of (Y, X) is known to the seller (Bayesian perspective)
- price discrimination better than uniform pricing
- X contains information about Y (correlated)

Data-based pricing

- seller does not know the joint distribution $F_{Y,X}$
- seller has access to an iid sample $(Y_i, X_i) : 1 \leq i \leq n$
- data-based uniform pricing: uses data to set a single price
- data-based price discrimination: uses data to set a pricing scheme for each value of x
- which one is better?

Main message

- data-based price discrimination is not necessarily better
- there is a trade-off

	theoretical revenue	learn distribution from data
price discrimination	higher	harder
uniform pricing	lower	easier

Main results

- minimax lower bounds for revenue deficiency of data-based pricing strategies
- price discrimination does have a slower rate for revenue generation
- devise data-based pricing strategies that achieves the minimax lower bounds

Literature

- theoretical computer science
- prior-independent (data-based) mechanism design
  - monopoly pricing: Huang, Mansour and Roughgarden (2018); Babaioff, Gonczarowski, Mansour and Moran (2018)
  - auctions: Cole and Roughgarden (2014); Dhangwatnotai, Roughgarden and Yan (2015); Guo, Huang and Zhang (2019); Devanur, Huang and Psomas (2016)
- optimal auctions with side information: Devanur et al. (2016)
  - requires that larger values of X are associated with larger values of Y in the sense of first-order stochastic dominance of conditional distributions

Examples of price discrimination

- marketing firms use a one-dimensional score of customer characteristics, response histories, zip code ...
- casinos use a one-dimensional score called the average daily win
- e-commerce

Roadmap

Revenue

- setting price to be p
- probability of transaction: $\mathbb{P}(Y > p) = 1 - F_Y(p)$
- revenue $R(p, F_Y) = p(1 - F_Y(p))$
- optimal uniform price p_U^* under F_Y

$$R(p_U^*, F_Y) = \sup_p R(p, F_Y)$$

Data-based uniform pricing strategy

- $\check{p}_U(\text{data}_Y)$
- maps valuations data $\{Y_i : i = 1, 2, \cdots, n\}$ to a single price
- expected revenue under F_Y

$$\mathbb{E}_{F_Y}[R(\check{p}_U(\text{data}_Y), F_Y)]$$

Minimax lower bound

- revenue deficiency: $R(p_U^*, F_Y) - \mathbb{E}_{F_Y}[R(\check{p}_U(data_Y), F_Y)]$
- minimax lower bound

$$\inf_{\check{p}_U} \sup_{F_Y \in \mathcal{F}^U} \left(R(p_U^*, F_Y) - \mathbb{E}_{F_Y}[R(\check{p}_U(data_Y), F_Y)]\right) \gtrsim n^{-2/3}$$

- $\mathcal{F}^U$ contains distributions with Lipschitz continuous density and concave revenue function
- the lower bound is for *any* data-based uniform pricing strategy

Empirical revenue maximization (ERM)

- ERM treats the empirical distribution as the true distribution
- $\hat{p}_U$: the optimal price that for the empirical cdf
- it has to be one of the observed Y_i's
- attaining the lower bound:

$$R(p_U^*, F_Y) - \mathbb{E}_{F_Y}[R(\hat{p}_U(data_Y), F_Y)] \lesssim n^{-2/3}$$

Roadmap

Revenue

- setting price to be $p(x)$ for covariate value $X = x$
- probability of transaction:
  $\mathbb{P}(Y > p(x)|X = x) = 1 - F_{Y|X}(p(x)|x)$
- revenue $R(p(\cdot), F_{Y,X}) = \int p(x)(1 - F_{Y|X}(p(x)|x))f_X(x)dx$
- optimal price discrimination p_D^* under $F_{Y,X}$

$$R(p_D^*, F_Y) = \sup_p R(p(\cdot), F_{Y,X})$$

Data-based price discrimination

- $\check{p}_D(x; \text{data})$
- maps entire data $\{(Y_i, X_i) : i = 1, 2, \cdots, n\}$ to a pricing function on the covariate space
- expected revenue under $F_{Y,X}$

$$\mathbb{E}_{F_{Y,X}}[R(\check{p}_D(\cdot; \text{data}), F_{Y,X})]$$

Minimax lower bound

- revenue deficiency:

$$R(p_D^*, F_{Y,X}) - \mathbb{E}_{F_{Y,X}}[R(\check{p}_D(\cdot; data), F_{Y,X})]$$

- minimax lower bound

$$\inf_{\check{p}_D} \sup_{F_{Y,X} \in \mathcal{F}^D} \Big(R(p_D^*, F_{Y,X}) - \mathbb{E}_{F_{Y,X}}[R(\check{p}_D(\cdot; data), F_{Y,X})] \Big) \gtrsim n^{-1/2}$$

- $\mathcal{F}^D$ contains distributions with Lipschitz continuous density and concave revenue function conditional on each covariate value

- the lower bound is slower than the uniform pricing case

K-markets empirical revenue maximization

- divide the covariate space equally into K markets
- implement ERM within each market
- attaining the lower bound: choosing $K \asymp n^{1/4}$,

$$R(p_D^*, F_{Y,X}) - \mathbb{E}_{F_{Y,X}}[R(\check{p}_D(\cdot; data), F_{Y,X})] \lesssim n^{-1/2}$$

Bias variance trade-off

- we are treating individuals in the same market as homogeneous
- small K: too few markets, the optimal price within the market is different from the pointwise optimal price, large bias
- large K: too many markets, less data for each market, large variance
- to balance, we need $K \asymp n^{1/4}$
- more sophisticated pricing strategies can at best improve the constant not the rate

Empirical study

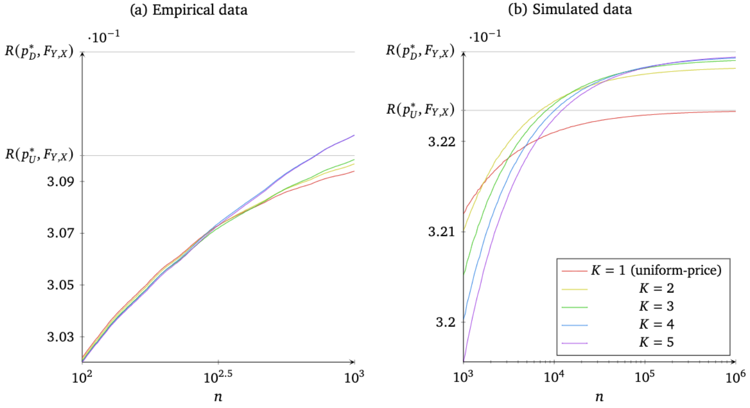
- eBay auction data set (Jank and Shmueli, 2010)
- sealed-bid second-price auction (bid = willingness to pay)
- 7-day auctions for Palm Pilot
- total observations 1203
- covariate: bidder rating on eBay

Simulation study

- $X \sim U[0,1]$
- $F(y|x) = y^{x+1}$

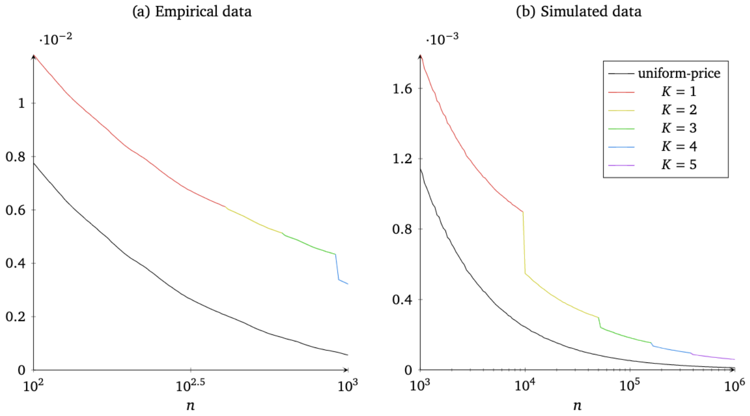
Expected revenue

Figure 1: Revenue under uniform and K-markets ERM strategies



Revenue deficiency

Figure 2: Data-based revenue deficiency under uniform and K-markets ERM strategies (with $K \asymp n^{1/4}$).



Conclusion

revenue difference between data-based PD and UP

$$
\begin{aligned}
&= \mathbb{E}_{F_{Y,X}}[R(\check{p}_D(\cdot; data), F_{Y,X})] - \mathbb{E}_{F_Y}[R(\check{p}_U(data_Y), F_Y)] \\
&= \underbrace{R(p_U^*, F_Y) - \mathbb{E}_{F_Y}[R(\check{p}_U(data_Y), F_Y)]}_{\asymp n^{-2/3}} \\
&- \underbrace{\left(R(p_D^*, F_{Y,X}) - \mathbb{E}_{F_{Y,X}}[R(\check{p}_D(\cdot; data), F_{Y,X})]\right)}_{\asymp n^{-1/2}} \\
&+ \underbrace{R(p_D^*, F_{Y,X}) - R(p_U^*, F_Y)}_{\text{theoretical difference, } \geq 0}
\end{aligned}
$$

Thank You!

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